

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch: Diploma (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{\theta} = \underline{\hspace{2cm}}$
 (A) 1 (B) 0 (C) 1/5 (D) 5
- b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \underline{\hspace{2cm}}$
 (A) e^2 (B) e (C) $e^{1/2}$ (D) None of these
- c) $\lim_{x \rightarrow 0} \frac{x^2 + x + 1}{x + 1} = \underline{\hspace{2cm}}$
 (A) 1 (B) 0 (C) -1 (D) None of these
- d) $\frac{d(\sqrt{x})}{dx} = \underline{\hspace{2cm}}$
 (A) $\frac{1}{2\sqrt{x}}$ (B) $\frac{1}{\sqrt{x}}$ (C) $-\frac{1}{x^2}$ (D) $2\sqrt{x}$
- e) $\frac{d(a^x)}{dx} = \underline{\hspace{2cm}}$
 (A) $a^x \log_e a$ (B) $\log_e a$ (C) $x^a \log_e a$ (D) $a^x \log_e x$
- f) $\frac{d(\cos x)}{dx} = \underline{\hspace{2cm}}$
 (A) $-\sec x$ (B) $\sec x$ (C) $-\sin x$ (D) $\sin x$
- g) $\frac{d(\cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$
 (A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$
- h) $\int 1 dx = \underline{\hspace{2cm}}$



(A) 0 (B) 1 (C) $x+c$ (D) None of these

i) $\int \tan x \, dx = \underline{\hspace{2cm}}$

(A) $\log \cos ecx + c$ (B) $\log \sec x + c$ (C) $\log |\cos ecx - \cot x| + c$

(D) $\log |\sec x + \tan x| + c$

j) $\int_1^3 \frac{2x}{1+x^2} \, dx = \underline{\hspace{2cm}}$

(A) $\log 5$ (B) $\log 2$ (C) $\log 10$ (D) $\log 20$

k) If $\mathbf{x} = (1, 1, 1)$ and $\mathbf{y} = (1, 0, 0)$ then $\mathbf{x} - \mathbf{y} = \underline{\hspace{2cm}}$.

(A) $(1,0,1)$ (B) $(0,1,1)$ (C) $(1,1,0)$ (D) $(0,0,1)$

l) $|2i + j - 3k| = \underline{\hspace{2cm}}$.

(A) $\sqrt{14}$ (B) $\sqrt{13}$ (C) $\sqrt{41}$ (D) 4

m) If $\bar{x} = 2i - j + k$ and $\bar{y} = i + 2j$ then $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$.

(A) 1 (B) 0 (C) -1 (D) -2

n) $(2, -1, 3) \times (-4, 2, -6) = \underline{\hspace{2cm}}$.

(A) $(-8, -2, -18)$ (B) $(8, 2, 18)$ (C) $(0, 0, 0)$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

a) If $\bar{a} = 2i + j - k$, $\bar{b} = i - j + 2k$ and $\bar{c} = i - 2j + k$ then find the direction cosines of $\bar{a} + \bar{b} - 2\bar{c}$.

(5)

b) If $\bar{a} = i + j + k$ and $\bar{b} = 2i - 2j + k$ then find unit vector perpendicular to \bar{a} and \bar{b} .

(5)

c) Evaluate: $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^{2x}}{x}$

(4)

Q-3

Attempt all questions

(14)

a) Evaluate: $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}}$

(5)

b) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$

(5)

c) Find $\frac{dy}{dx}$ if $y = e^x \sin x \cos x$.

(4)

Q-4

Attempt all questions

(14)

a) $S = t^3 - 6t^2 + 8t - 4$ gives the distance travelled by a body in t seconds. Find velocity and acceleration at $t = 4$ second.

(5)

b) Find $\frac{dy}{dx}$ if $y = \log \sqrt{\frac{a+x}{a-x}}$.

(5)

c) If $\bar{x} = (-4, 9, 6)$, $\bar{y} = (0, 7, 10)$, $\bar{z} = (-1, 6, 6)$ then show that

(4)

$$(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$$

Q-5

Attempt all questions

(14)



a) Prove that angle between two vectors $i + j - k$ and $2i - 2j + k$ is $\sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right)$. (5)

b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. (5)

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$ (4)

Q-6

Attempt all questions (14)

a) Forces $(1, 2, 3)$, $(-1, 2, 3)$ and $(-1, 2, -3)$ act on a particles and the particle moves from the point $(0, 1, -2)$ to $(-1, 3, 2)$. Find the work done by the forces. (5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$. (5)

c) Find derivative of $y = x^3 - 2x$ using first principle. (4)

Q-7

Attempt all questions (14)

a) Evaluate: $\int \frac{(1-3x)^2}{x^3} dx$ (5)

b) Find the area of circle $x^2 + y^2 = r^2$. (5)

c) For what value of R , vectors $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other? (4)

Q-8

Attempt all questions (14)

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. (5)

b) If $y = A \cos pt + B \sin pt$ then prove that $\frac{d^2y}{dt^2} + p^2y = 0$. (5)

c) Evaluate: $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ (4)

